Geometric Correction

Operations intended to restore or compensate the distortions of an image in geometry.

Sources of geometric distortions:

- Sensor characteristics
- Viewing geometry
- Platform motion
- Target motion

Distortion Sources

A. Sensor Characteristics

- optical distortion
- aspect ratio
- non-linear mirror velocity
- detector geometry & scanning sequence
  aircraft/satellite motion (pitch, roll, yaw)

B. Viewing Geometry

- panoramic effect
- earth curvature

C. Platform Motion

- attitude changes
- position variations

D. Target Motion

- earth rotation
- moving targets

Symptoms

Geometric Distortions usually appear as:

- changes of scale over the image
- irregularities in the angular relationships among the image elements
- displacement of objects in an image
- occlusion of one image element by another

Corrections

The nature of the "correction" depends upon
the ultimate use of the data:

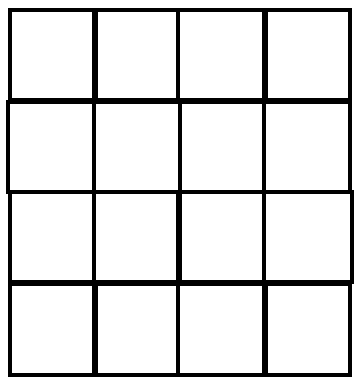
- area measures ==> equal area projection
- shape measures ==> projection that preserves the angular relationships of the scene

Correcting the distortions is often costly

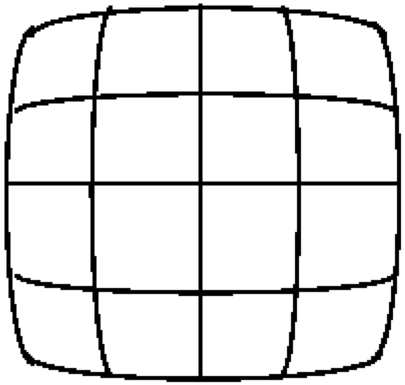
- computer & operator time
- affects spatial and radiometric resolution

Sensor Characteristics 1: Optical Distortion:

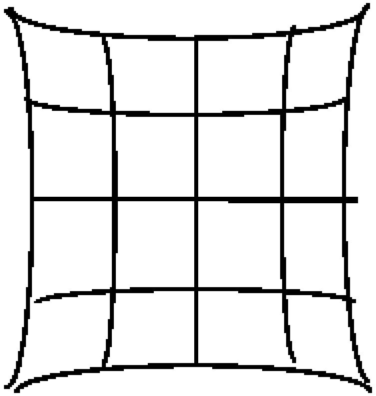
Note: This type of distortion is not usually a serious problem in remote sensing systems.



original



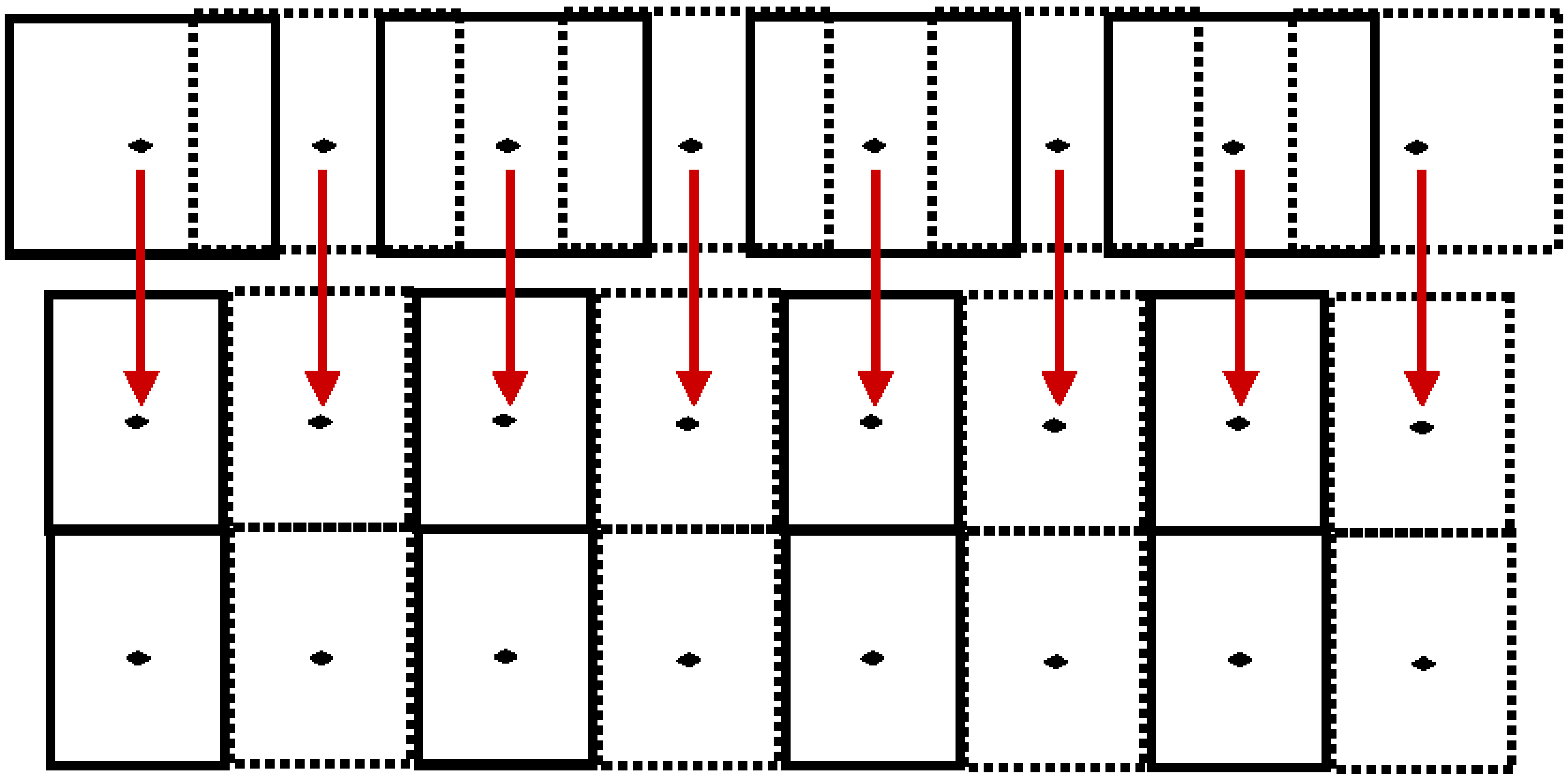
negative (barrel) distortion



positive (pincusion) distortion

These distortions are radially symmetric and characteristic of the optical system.

Sensor Characteristics 2: Aspect Ratio:



If displayed in square pixels system, the image is distorted.

Sensor Characteristics 3: Non-linear mirror velocity:

- Uniform pixel spacing along a scan line presumes that the mirror velocity is constant.
- An oscillating mirror (MSS, TM) must stop at the end of each scan and reverse direction.

Sensor Characteristics 4: Detector geometry and scanning sequence

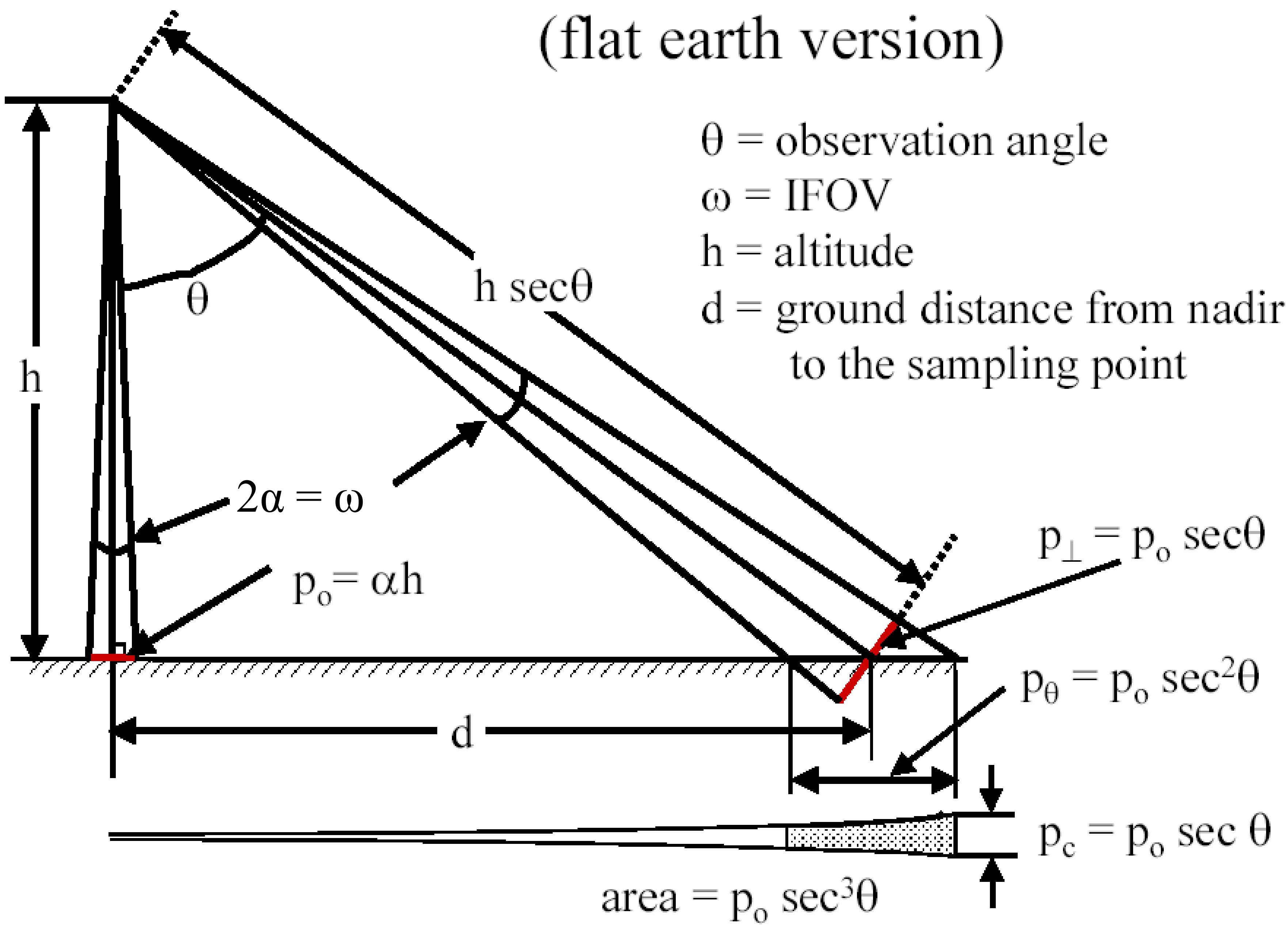
Presumptions for a regular sampling pattern:

a) the detectors are all exactly in the focal plane

b) the scanning sequence and timing will exactly overlay detectors for different spectral bands

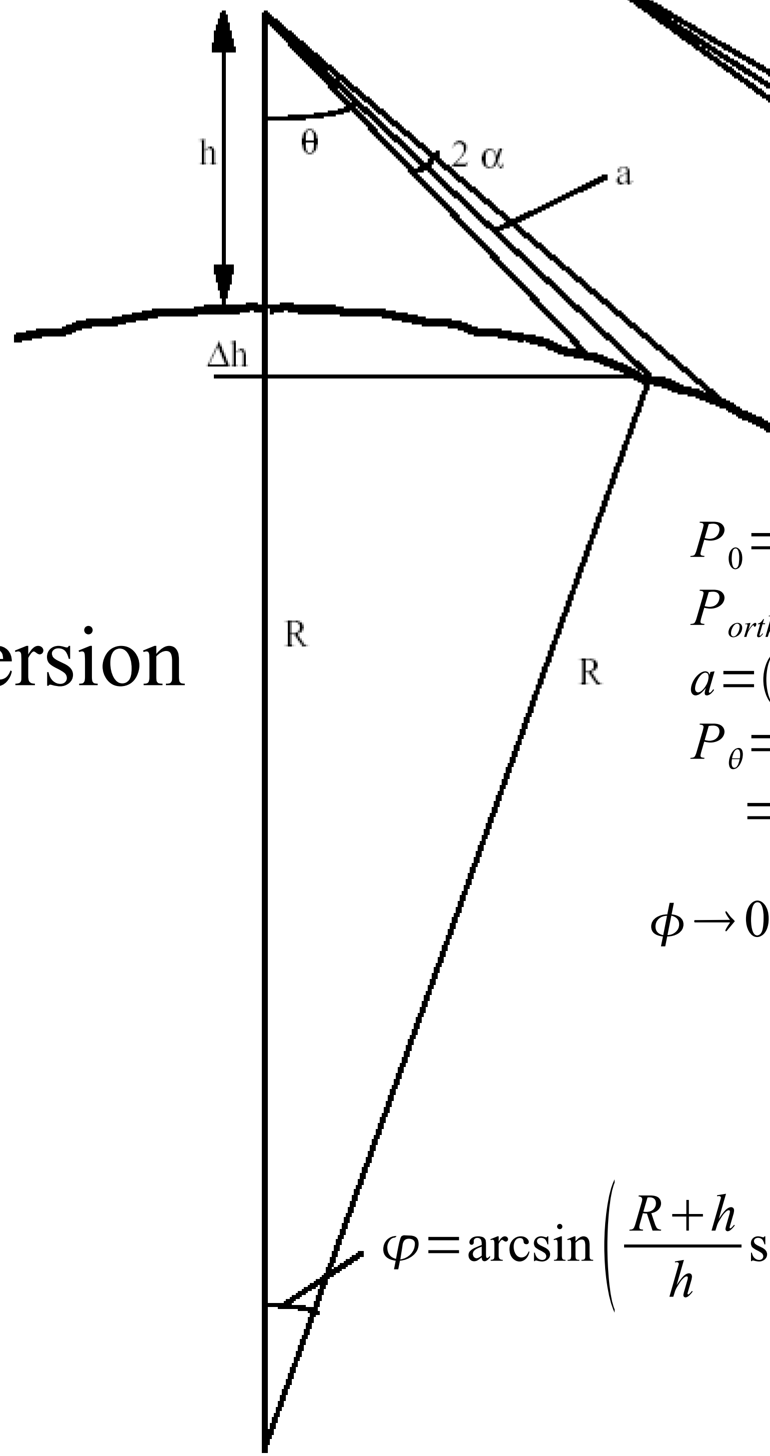
Viewing Geometry: panoramic effect

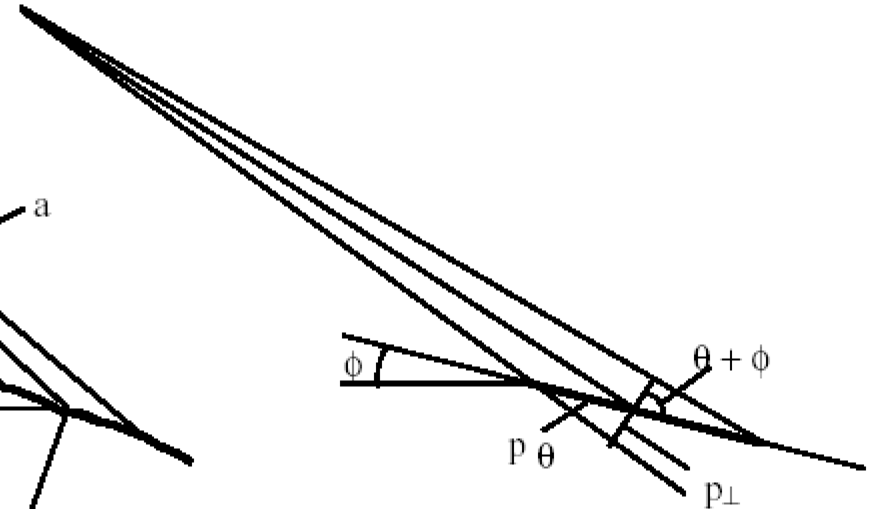
a scale distortion (an increase in cell size) for observations away from nadir due to an increase in the area covered by the sensor.



Viewing Geometry: panoramic effect

Curved earth version



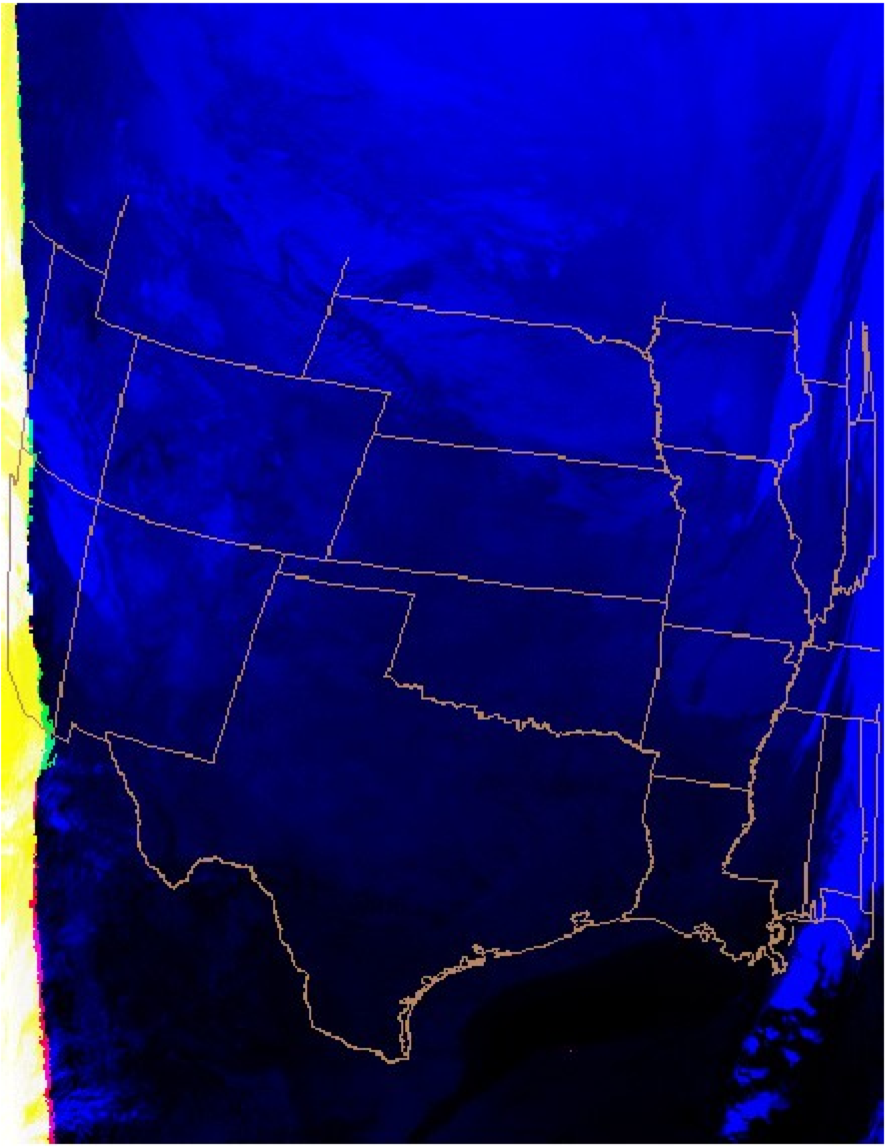


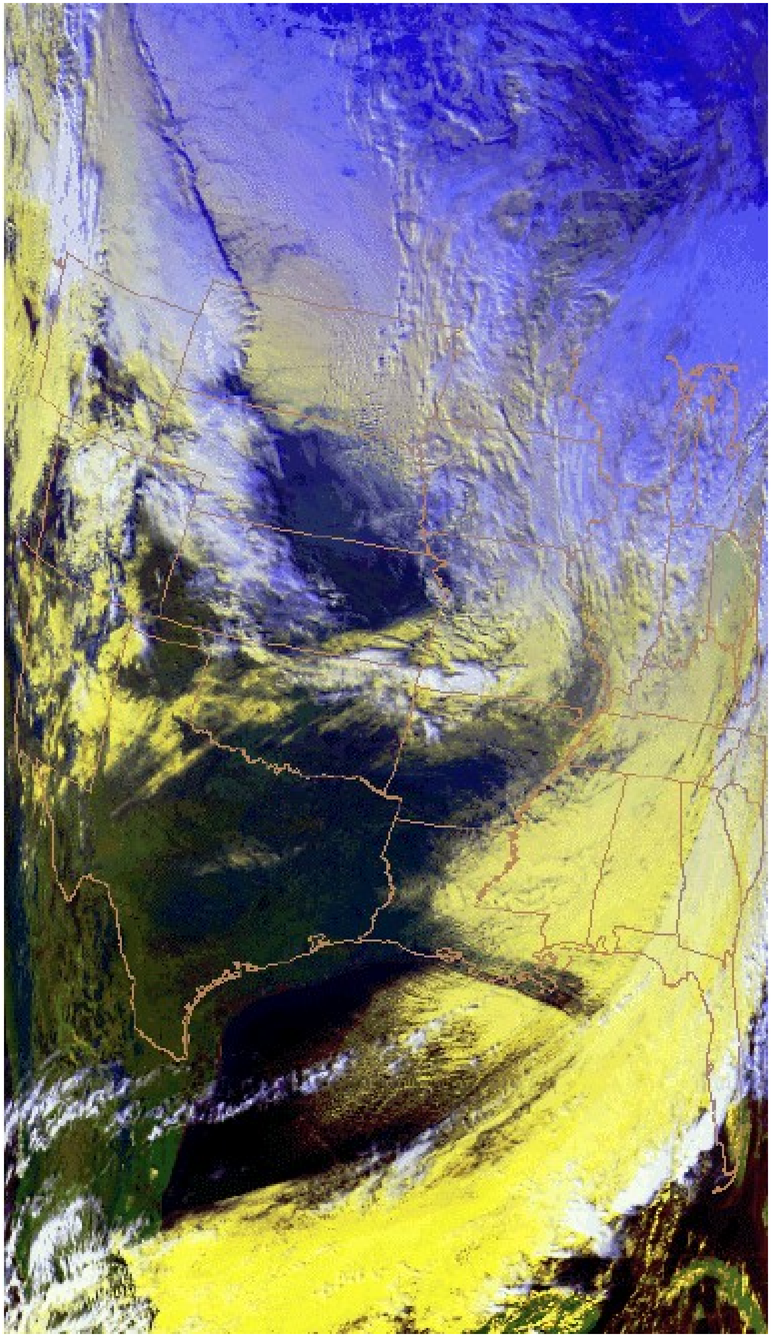
$$P_0 = h\alpha$$
$$P_{ortho} = a\alpha$$
$$a = (h+\Delta h)\sec(\theta)$$
$$P_\theta = P_{ortho} + \sec(\theta+\phi)$$
$$= (h+\Delta h)\alpha\sec(\theta)\sec(\theta+\phi)$$

$$\phi \rightarrow 0 \quad \cos(\theta+\phi) \rightarrow \cos(\theta)$$
$$\Delta h \rightarrow 0$$
$$P_\theta \rightarrow P_0 \sec^2(\theta)$$

$$\varphi = \arcsin\left(\frac{R+h}{h}\sin(\theta)\right) - \theta$$

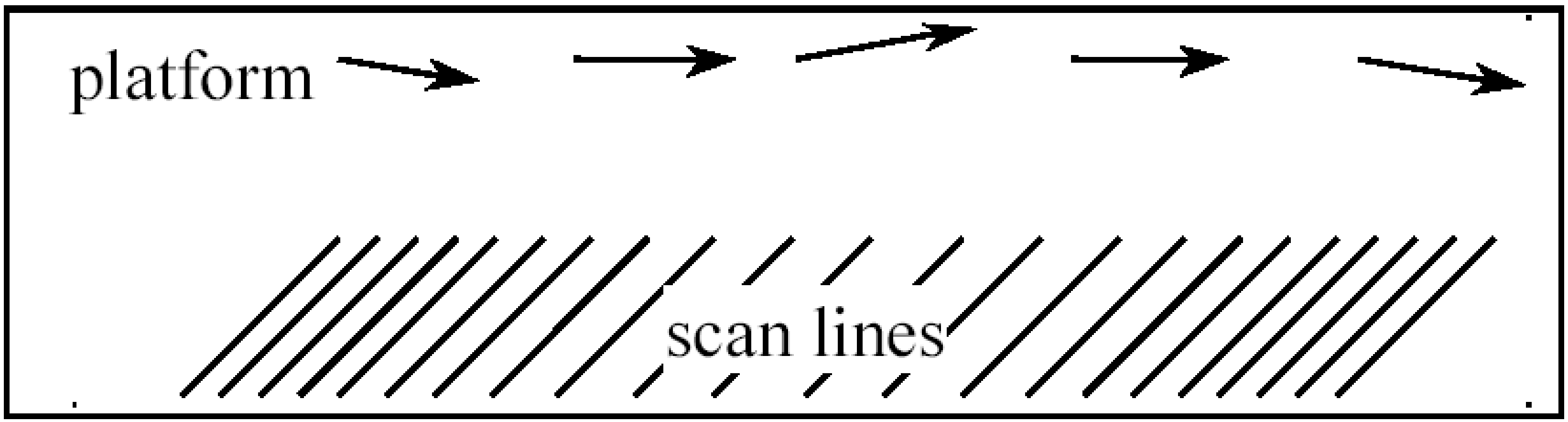
panoramic effect examples





Platform motion: Attitude changes

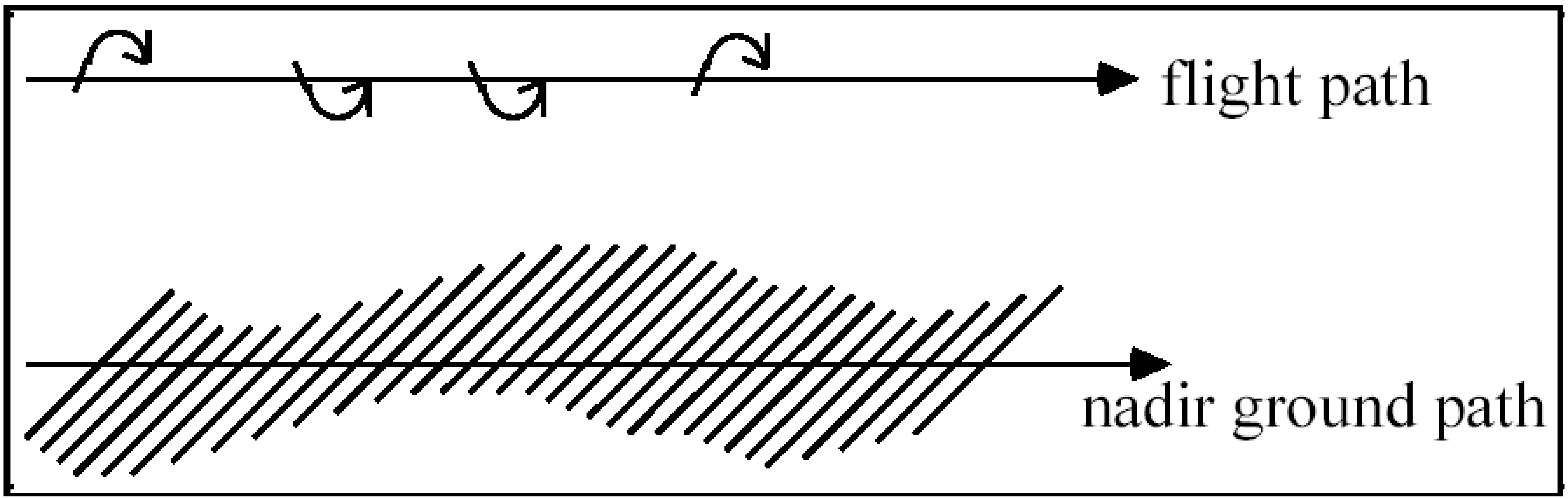
Pitch: "Vertical rotation of a sensor platform, in the 'nose up' plane."



Changes in pitch will result in changes in the spacing of the scan lines.

Platform motion: Attitude changes (cont.)

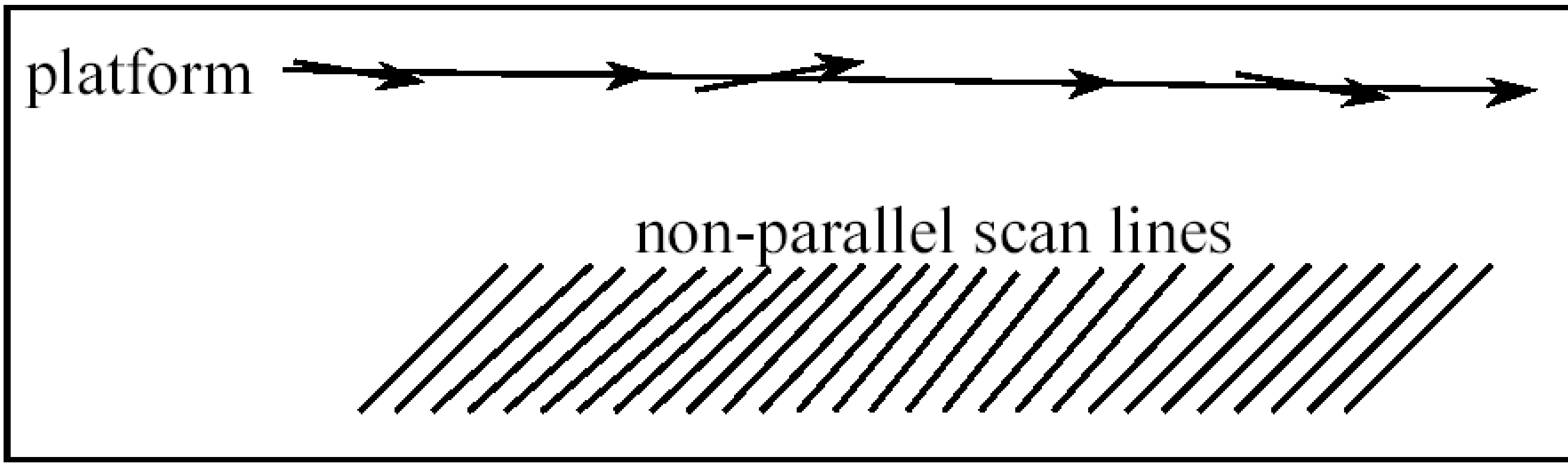
Roll: "Rotation of a sensor platform around the flight vector, hence in a "wing down" direction."



Roll causes lateral shifts in the scan line position.

Platform motion: Attitude changes (cont.)

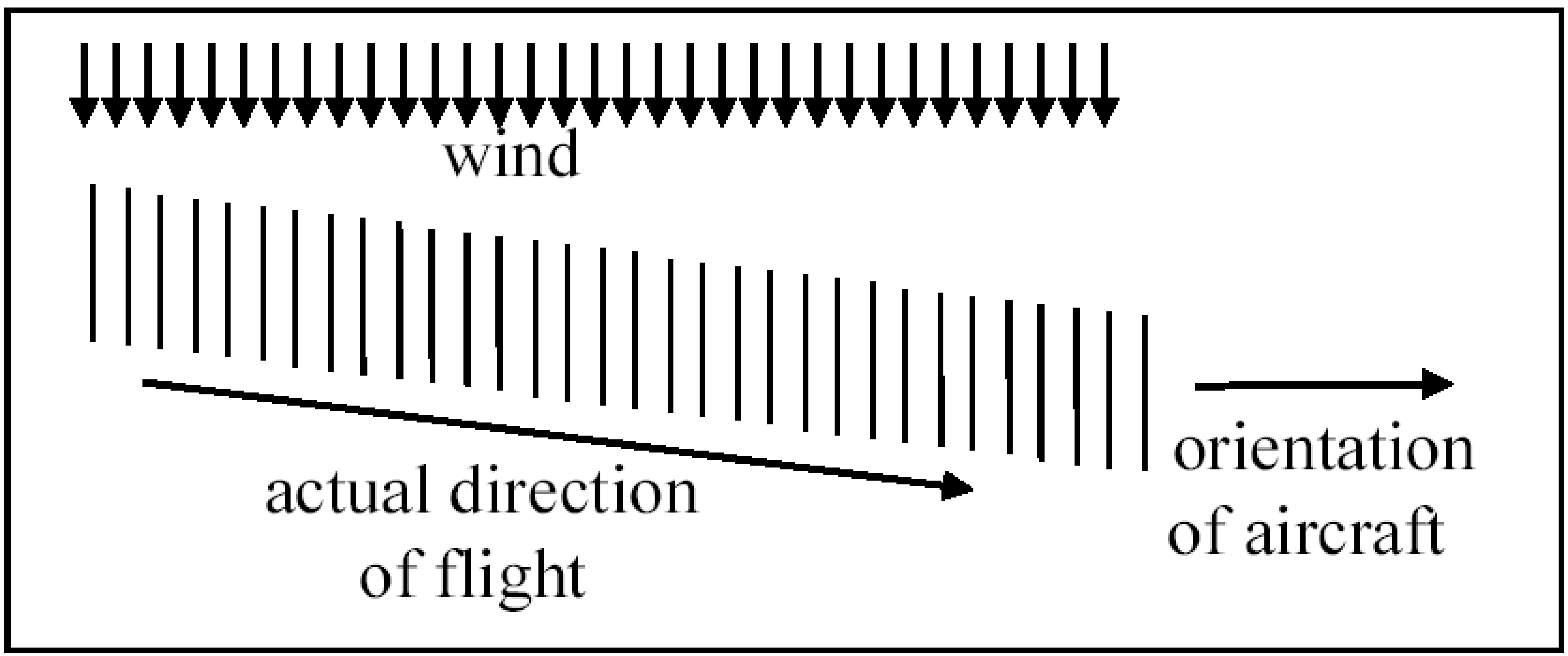
Yaw: "Rotation of a sensor platform in the horizontal plane, or about its vertical axis, hence in a "nose right" direction."



Changes in yaw will result in scanlines that are not parallel.

Platform motion: position changes

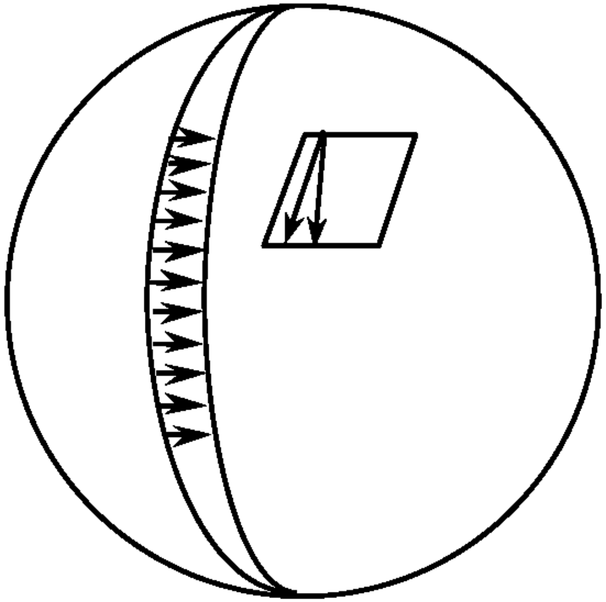
1. altitude - results in variations in scale
2. slew - motion of the aircraft or satellite perpendicular to the intended direction of motion



Target motion

- Distortion will depend entirely on the nature of that motion relative to the sampling rate & sequence of the imaging system.
- When the motion of the scene is of the same order as the sampling rate the image will be blurred.
- A photo of a nearby building taken from the side window of a car moving at 50 mph when the shutter speed is 1/60 second, will result in a blurred image

Target motion : earth rotation



- The rotation of the earth is slow relative to the sampling rate of the Landsat MSS (~ 0.4 μs/pixel) and it is even slow relative to the scan rate (~33 ms/scan). Thus, there is no obvious blur in the final image.
- However, between the time that the first scan of a Landsat MSS image and the time of the last scan, the earth will have rotated a significant distance relative to the size of a resolution element.

"Exact" Geometric Corrections

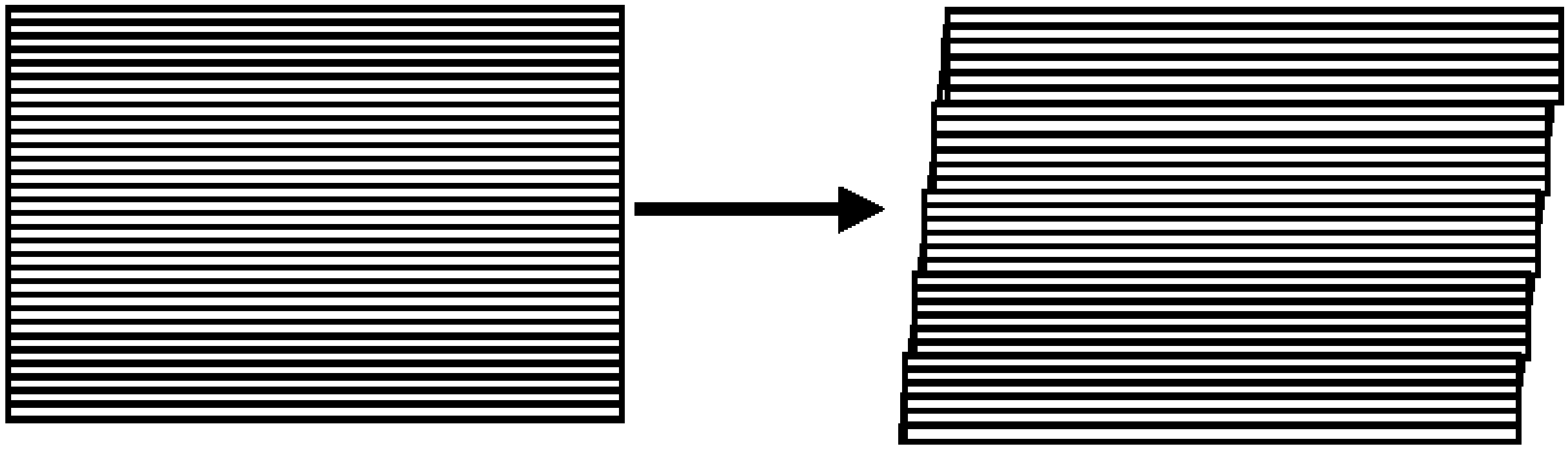
When enough is known about the source of the geometric distortions, it may be possible to ***approximate*** an ideal correction.

Example: Earth rotation

a) Displacement is nearly perpendicular to the flight path (along the scan line).

b) Rate of displacement is related to the orbital velocity and the angular velocity of the earth (predictable)

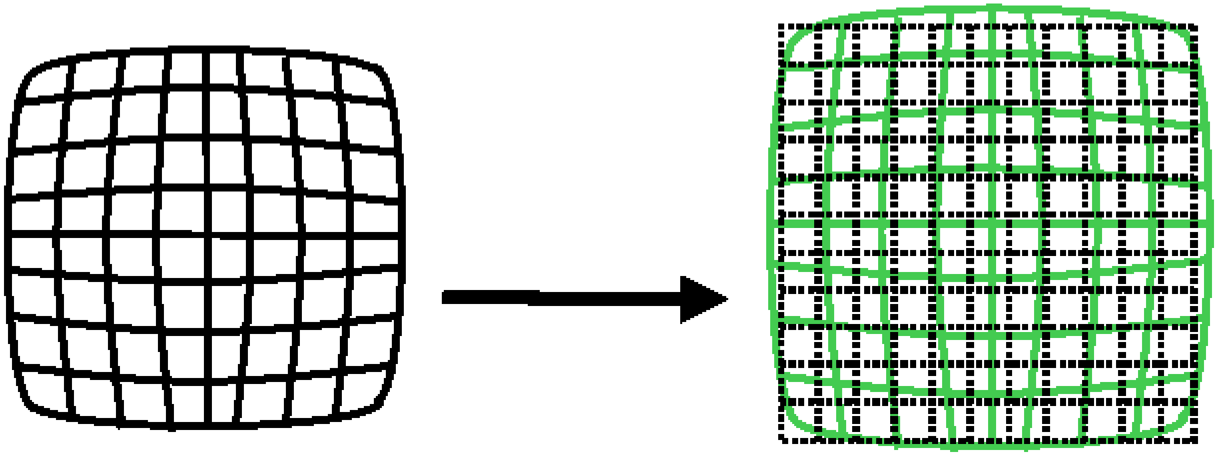
(Ideal) Earth Rotation Correction



- only shifts by integral units of the sampling interval are allowed.
- no resampling (other than the pixel shift) is required.

(Ideal) Optical Distortion Correction

- Displacement of each pixel is radial.
- Amount of displacement is defined by the optics of the system.



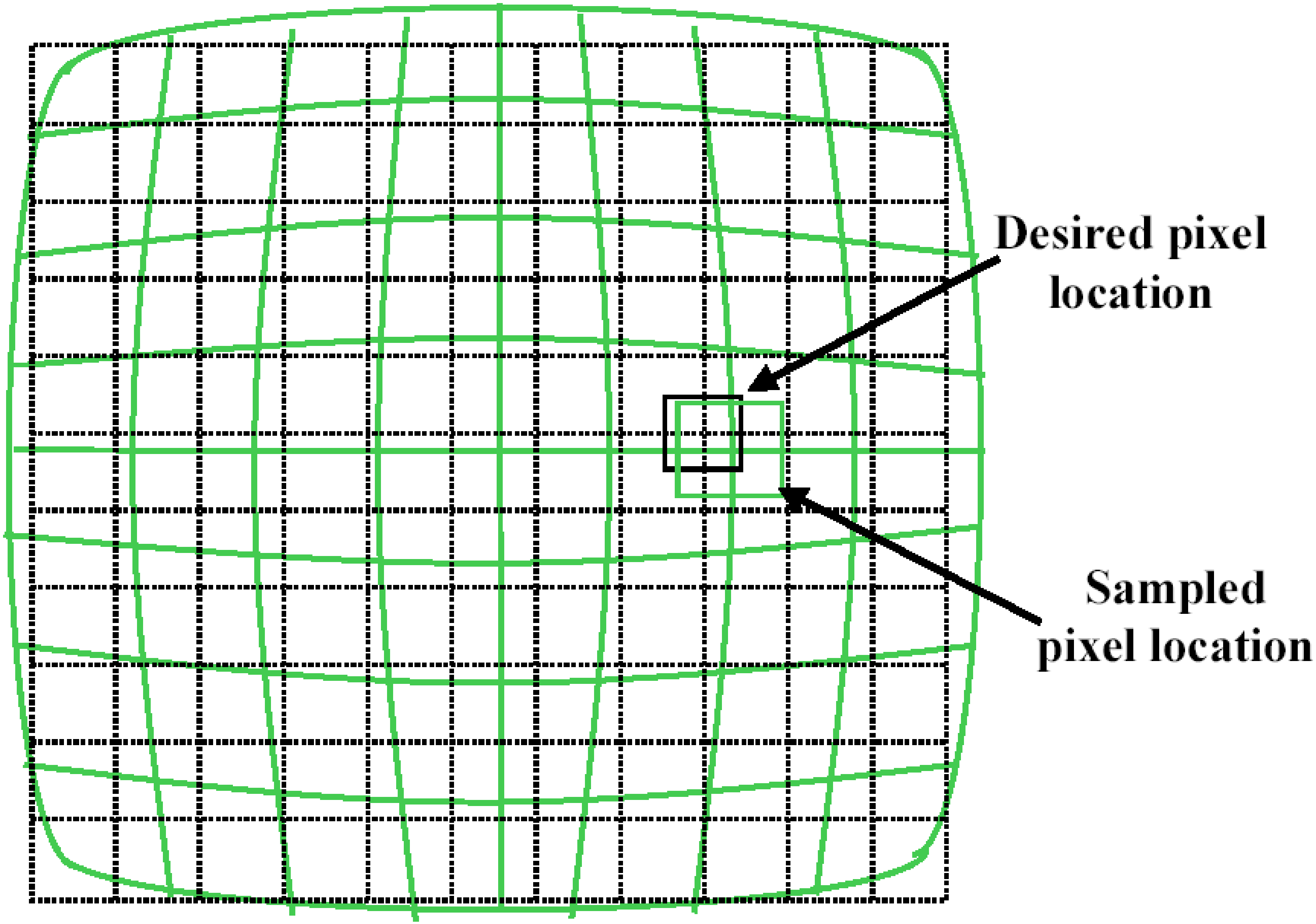
a) the amount of the shift is proportional to the distance from the optical axis

b) the shift direction varies over the image

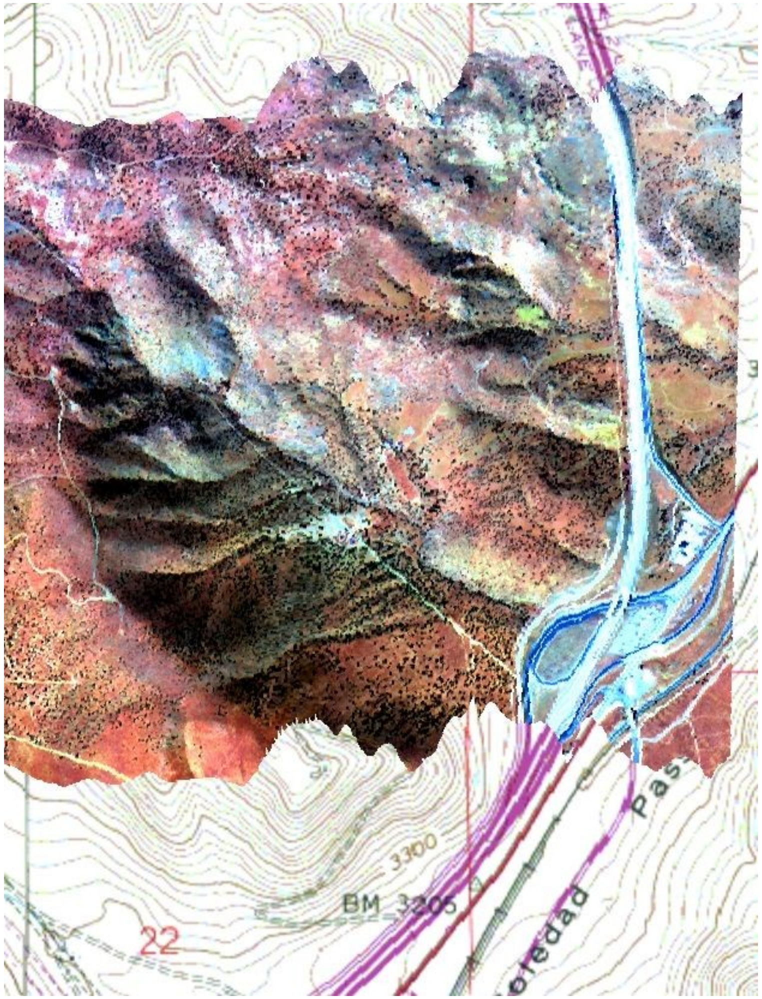
c) requires shifts by non-integral units of the sampling interval

d) requires resampling

(Ideal) Optical Distortion Correction



Airborne Imagery Orthorectification



Airborne Imagery Orthorectification

CASI airborne hyperspectral imagery

Different Levels of Satellite RS Products

Level 1A	Raw image (relative radiometric correction only)
Level 2	Systematic geometric correction
Level 2C	Systematic geo-correction (but also corrected for mosaic from across strips)
Level 3	Rigorous geo-correction (w/ GCPs but w/o DTM)
Level 4	Orthorectified (w/ GCPs & DTM)
Fusioned	Using Level 2C or higher

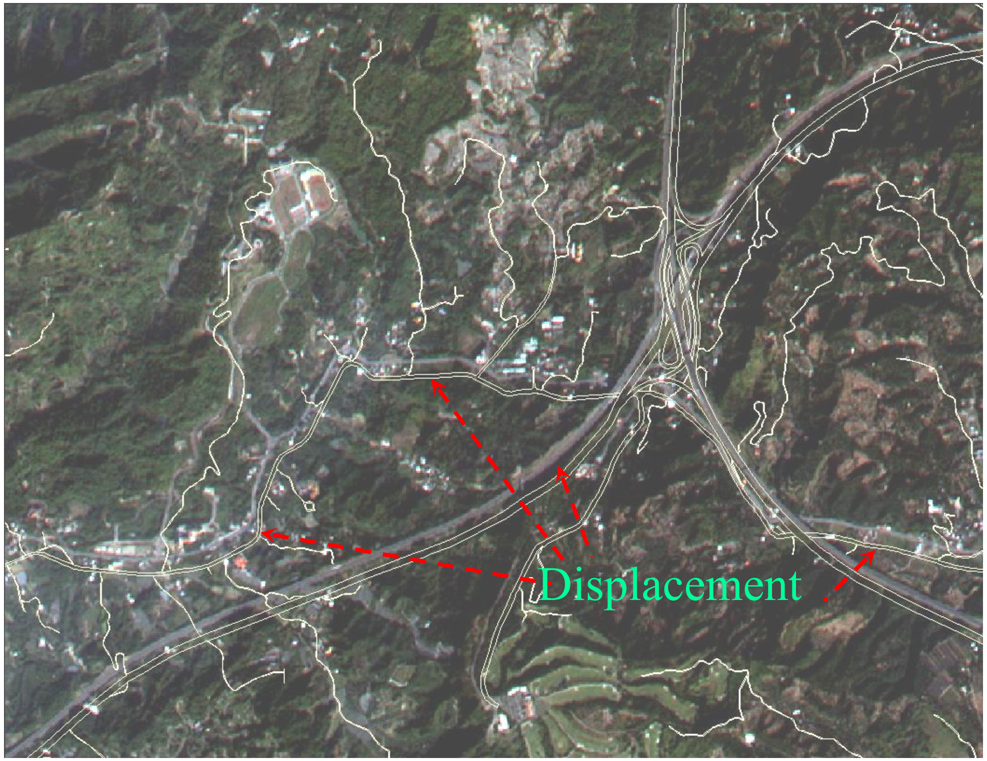
Comparison of Different Levels

L1A

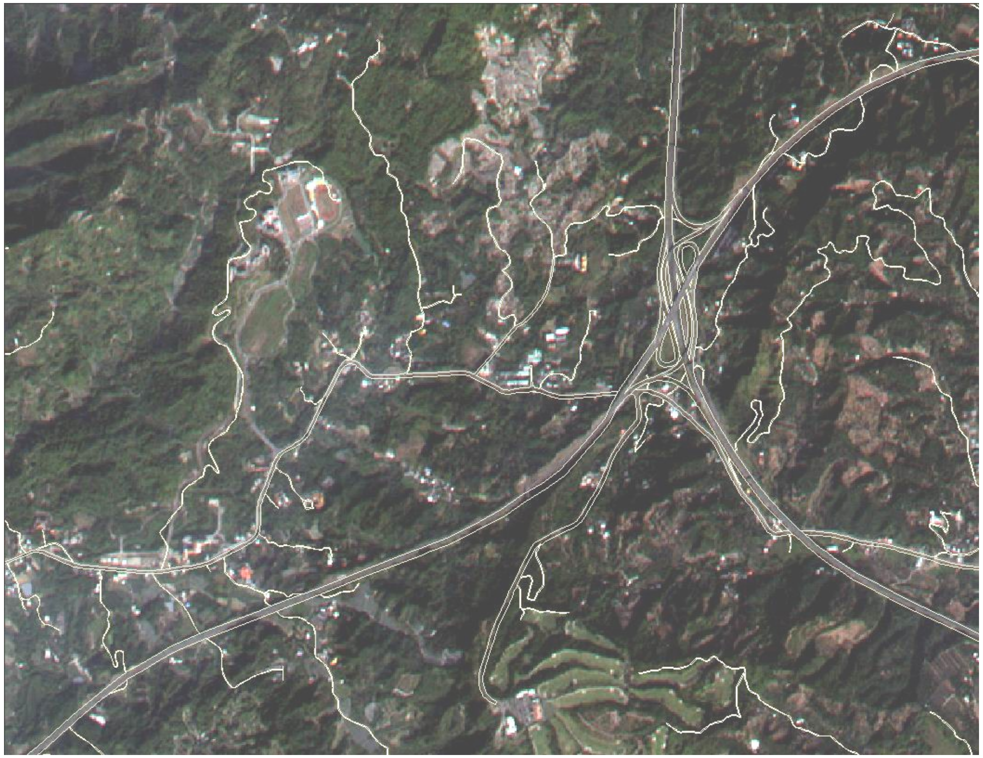
L2

L4

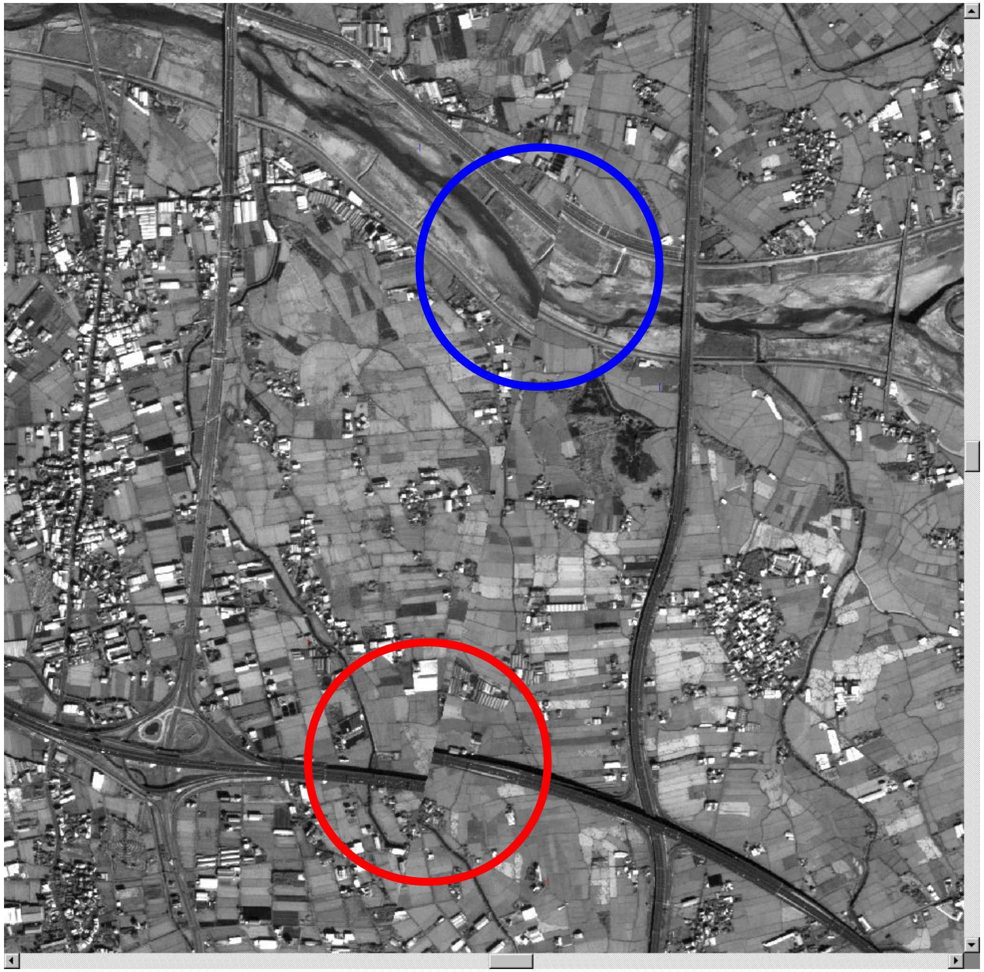
Overlay with 1:5000 Vector Data



Level 2



Level 4



Level 2

Level 2C

Correction by Rational Function Model (RFM)

- No ephemeris data required
- Simple and fast to carry out
- Less ground control points required
- May be difficult to achieve high level orthorectification

$$r = \frac{p_1(X,Y,Z)}{p_2(X,Y,Z)} = \frac{\sum_{i=0}^{m1}\sum_{j=0}^{m2}\sum_{k=0}^{m3} a_{ijk} X^i Y^j Z^k}{\sum_{i=0}^{m1}\sum_{j=0}^{m2}\sum_{k=0}^{m3} b_{ijk} X^i Y^j Z^k}$$

$$c = \frac{p_3(X,Y,Z)}{p_4(X,Y,Z)} = \frac{\sum_{i=0}^{m1}\sum_{j=0}^{m2}\sum_{k=0}^{m3} c_{ijk} X^i Y^j Z^k}{\sum_{i=0}^{m1}\sum_{j=0}^{m2}\sum_{k=0}^{m3} d_{ijk} X^i Y^j Z^k}$$

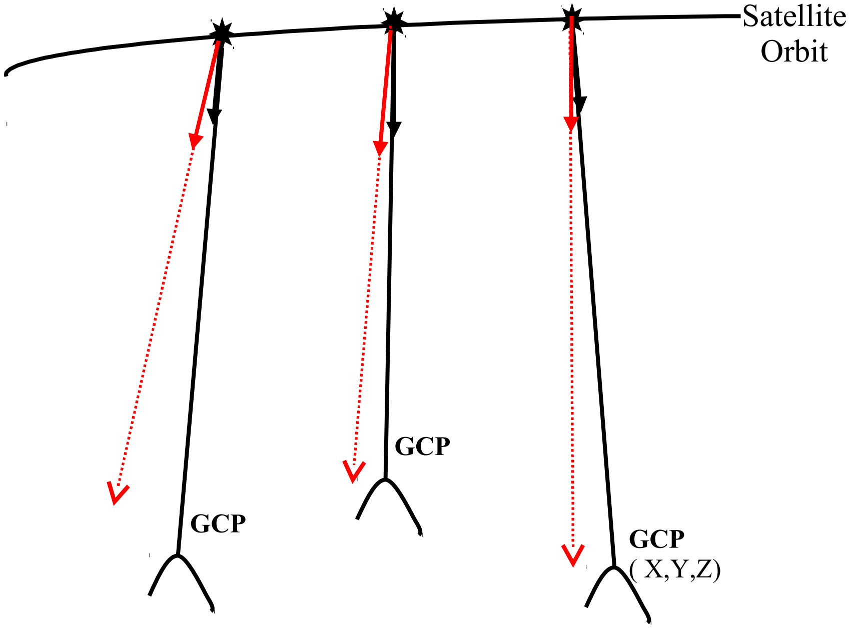
Rigorous Correction (Orthorectification)

- Require ephemeris
- Require more ground control points
- Complex mathematics
- Can achieve high level orthorectification

Procedure

- Position and Attitude Correction
  - Initial conditions
  - Attitude correction
  - Position correction
  - Least-squares collocation
- Orthorectification
  - Back projection
  - Resampling

Attitude Correction



$$\vec{G} - \vec{P} = S \cdot M_z(\kappa_t) \cdot M_y(\phi_t) \cdot M_x(\omega_t) - \vec{U}$$

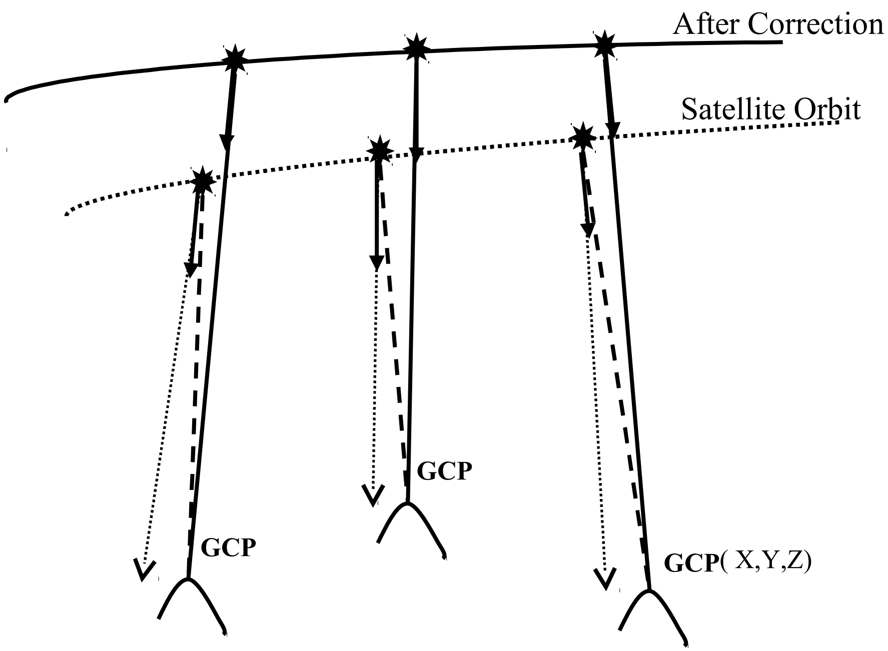
$$\omega_t = \omega_0 + \omega_1 t + \omega_2 t^2 + \omega_3 t^3$$

$$\phi_t = \phi_0 + \phi_1 t + \phi_2 t^2 + \phi_3 t^3$$

$$\kappa_t = \kappa_0 + \kappa_1 t + \kappa_2 t^2 + \kappa_3 t^3$$

M_x, M_y, M_z:	rotation matrix
$\omega_t, \phi_t, \kappa_t$:	attitude angles
$\omega \sim \kappa_{0\sim3}$:	coefficients
t:	time

Position (Orbit) Correction



$$\vec{G}-(\vec{P}+\Delta\vec{P}_t)=S\,\vec{U}\,'$$

$$\Delta\vec{P}_t=\begin{bmatrix} X_0+X_1t \\ Y_0+Y_1t \\ Z_0+Z_1t \end{bmatrix}$$

$U\,'$: Observation after correction

Least-Squares Adjustment

1. Objective: eliminate local errors
2. How: $D_k = \vec{v}_k [\Sigma_k]^{-1} \vec{\varepsilon}_k$

k : x, y, z

D_k : Adjustment of targets in k direction

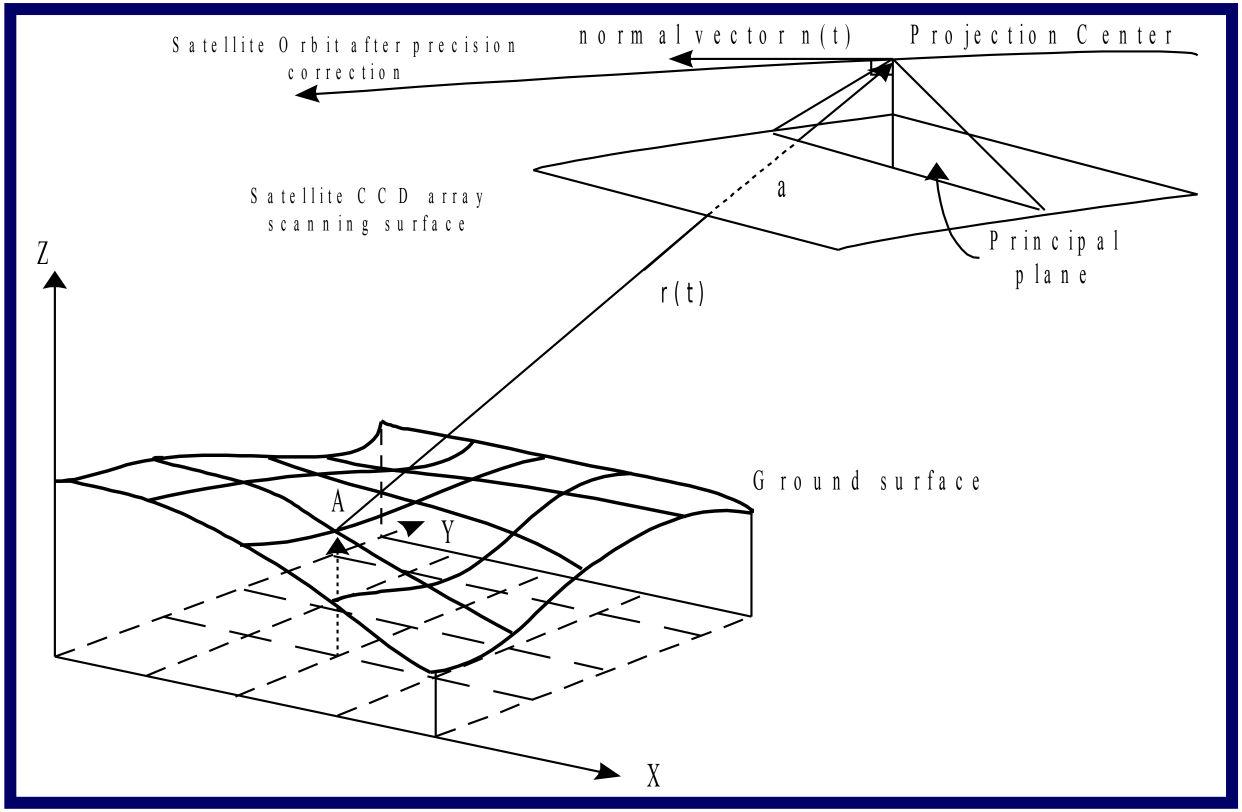
v_k : Covariance between targets and GCPs

Σ_k : Covariance matrix of GCPs

ε_k : Residual of GCPs

Backward Projection

From world coord. To image coord.

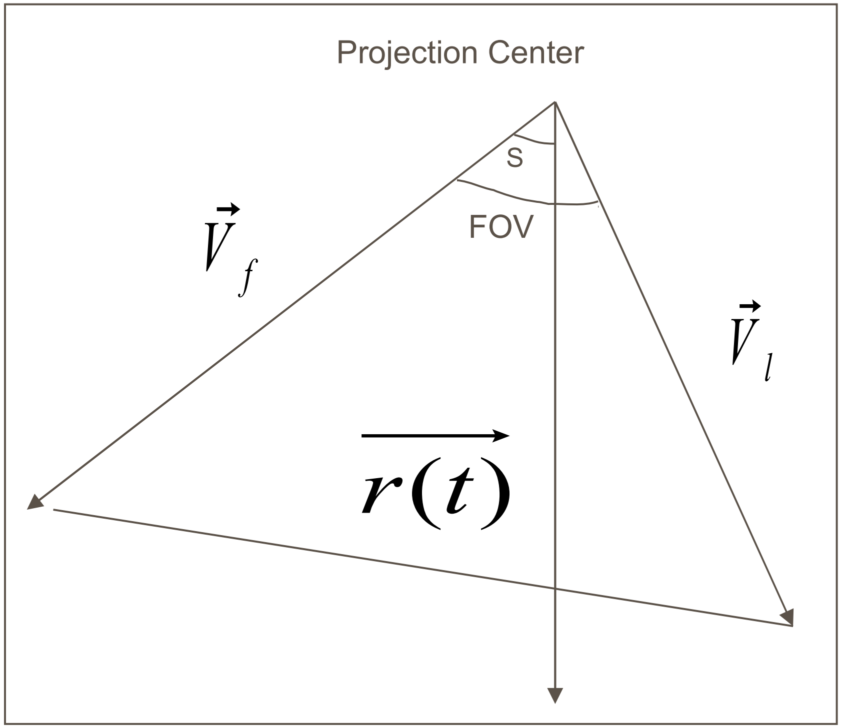


- r(t)=vector from A to projection center at anytime, t
- n(t)=normal vector of principal plan
- f(t)=r(t) · n(t)=0
- Use Newton-Raphson iteration to solve t

Backward Projection (cont.)

$$\text{Line} = (t - t_0) / integ\_time$$

$$\text{Sample} = (s / FOV) / N$$



Simple Geometric Correction Procedures

1. Select the appropriate projection or reference map (or image).

   registration: simple point-to-point match of an image to another image or map,
   rectification: correcting an image to a specific map projection.

2. Select a regular grid which fits the desired projection (i.e., determine the spacing and position of the grid points.)

   NOTE: Step 1 and 2 depend on individual applications.

Geometric Correction Procedures (cont.)

3. Select a set of "ground control points" (GCP's) – pixels whose locations can be determined accurately in the base map and the image.

4. Define the transformation and compute the positions of the reference grid points in the image coordinate system. The transformation should deform the grid in such a way that the average distance between GCP pixels and their map locations is minimized.

5. Resample the image data in order to assign gray values to each grid point.

Map Projections

There is no perfect map projection.

Projections may preserve

area

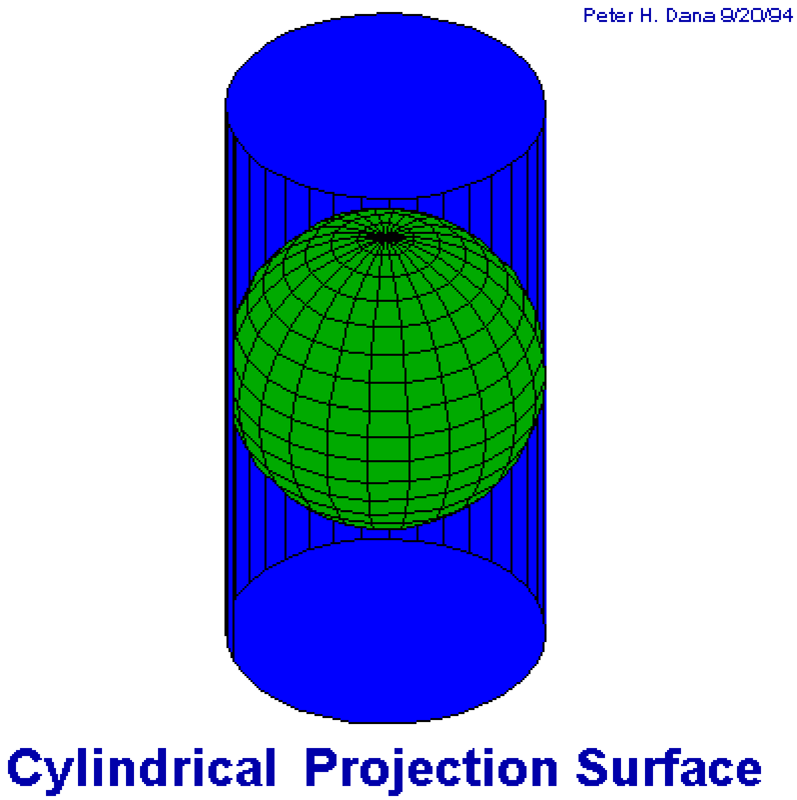
or *shape*

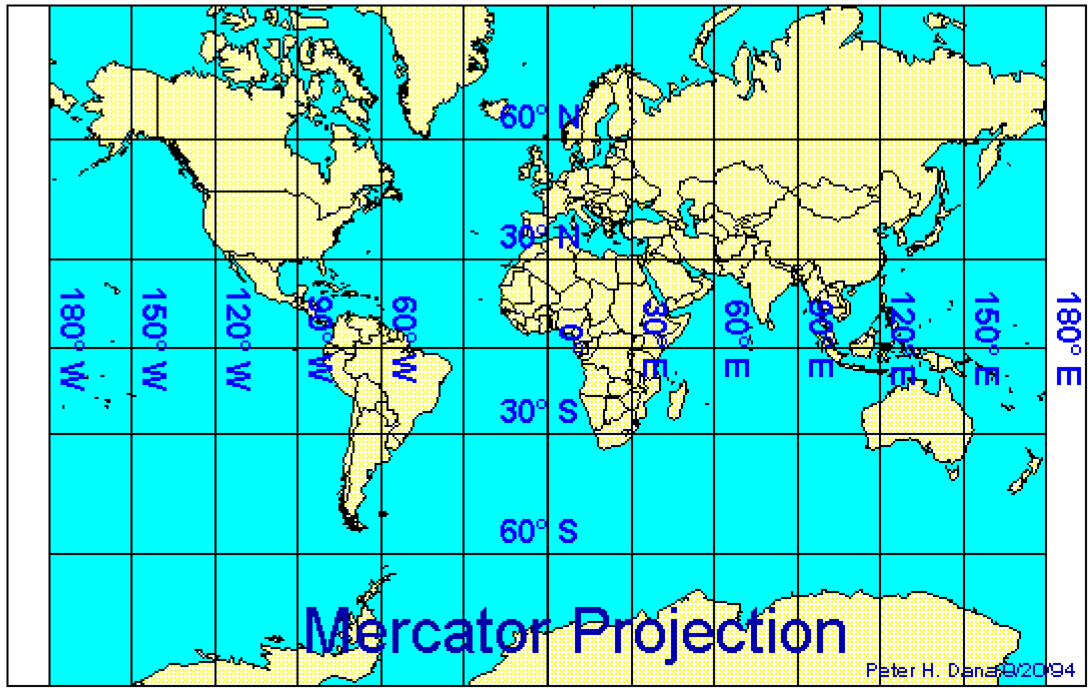
or *direction*

or *distance*

For small enough areas, all factors may be preserved within the precision of the sampling.

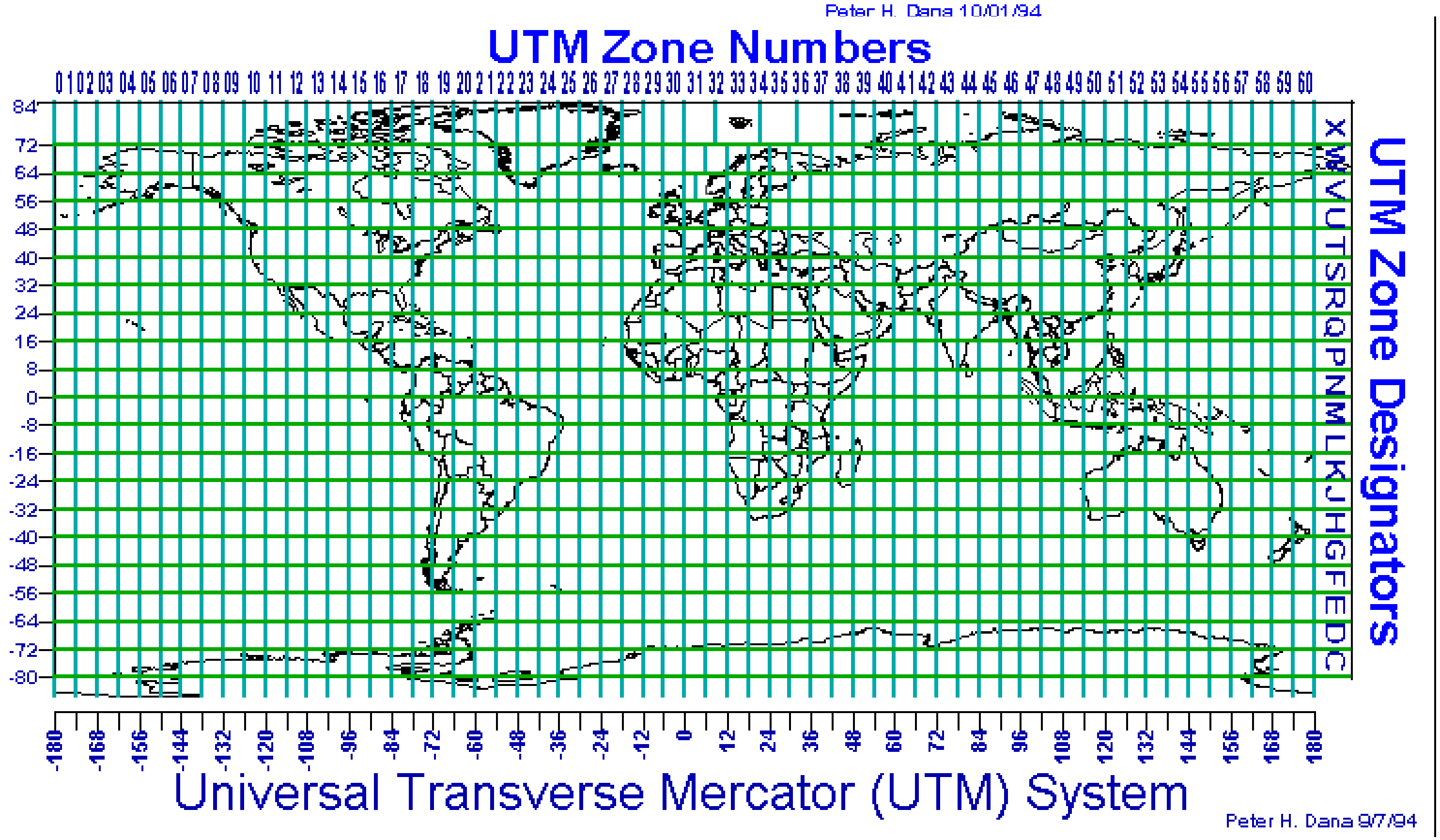
Mercator Projection





Map Projections, by P. H. Dana, 2004
http://www.colorado.edu/geography/gcraft/notes/mapproj/mapproj_f.html

Universal Transverse Mercator (UTM) Projection



Ground Control Points (GCP's)

Pixels whose locations can be determined accurately in the base map and the image, used to create the mapping of the image to the base map.

For each control point the pixel coordinate must be matched with the coordinate of the control point in the desired coordinate system.

Selection of Ground Control Points

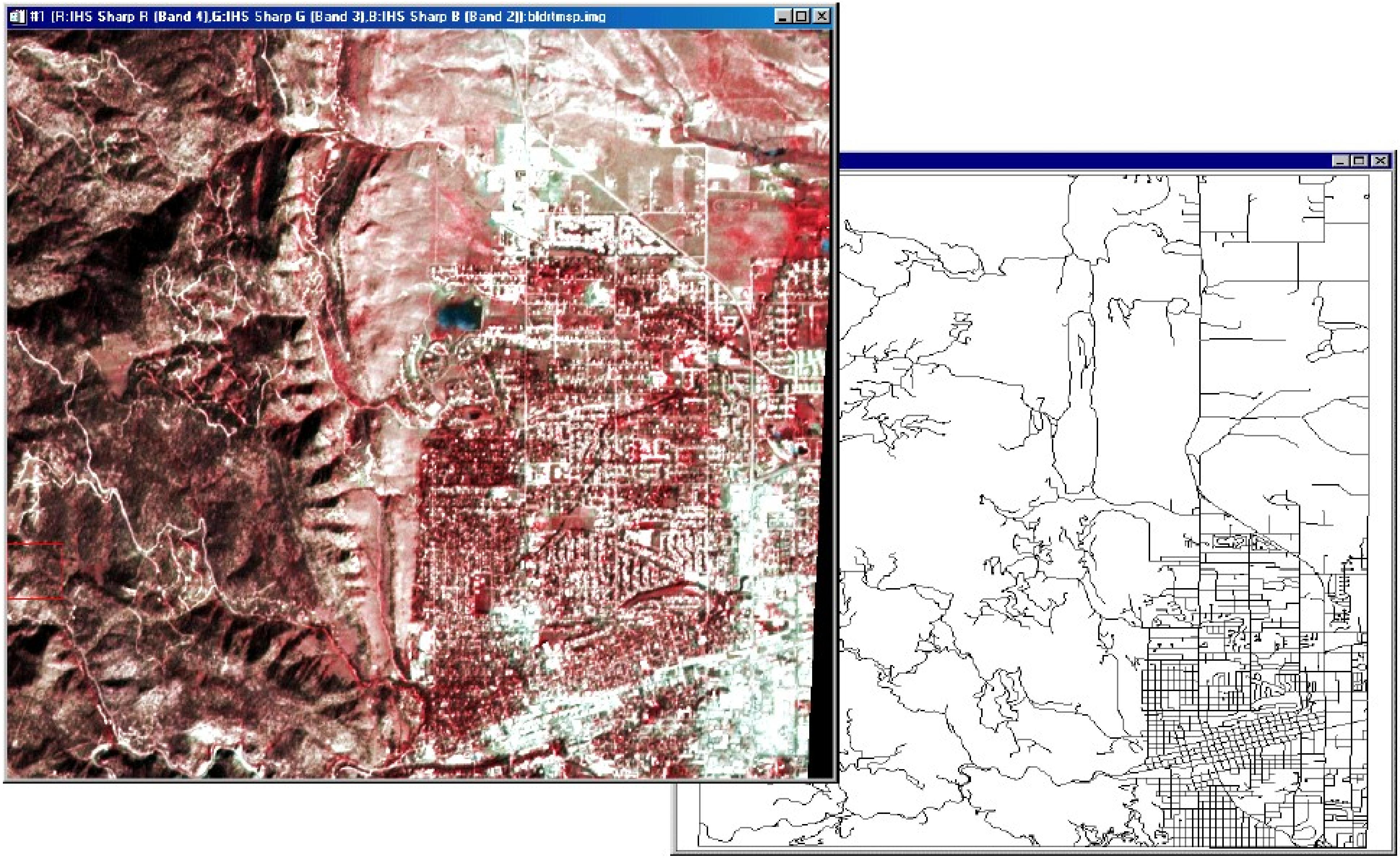
Pixels used for GCP's should be:

- easy to locate accurately on the base map.
- easily recognizable features or landmarks in the image.

Examples;

- crossroads
- land/water boundaries
- cultural features (airstrips, buildings, etc.)

Image-to-Map registration example:



Geometric Transforms Procedure

1. Select a set of GCP's
2. Select a transformation (linear, 2nd order, 3rd order, ...)
3. Determine the coefficients that will minimize the error (RMSE).
4. Analyze the residual errors.
5. Adjust the selection and placement of GCP's.
6. Repeat steps 3-5 until the error is acceptable.

Linear Transformation

Translation	$x' = x + x_0$	$y' = y + y_0$
Rotation	$x' = y\sin\theta + x\cos\theta$	
	$y' = y\cos\theta - x\sin\theta$	
Scaleing	$x' = mx$	$y' = ny$

General linear transformation:

$$x' = a_0 + a_1 x + a_2 y$$

$$y' = b_0 + b_1 x + b_2 y$$

Non-Linear Transformations

Typically non-linear, polynomial fit such that:

$$x' = a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2 + \ldots$$

$$y' = b_0 + b_1x + b_2y + b_3xy + b_4x^2 + b_5y^2 + \ldots$$

Resampling

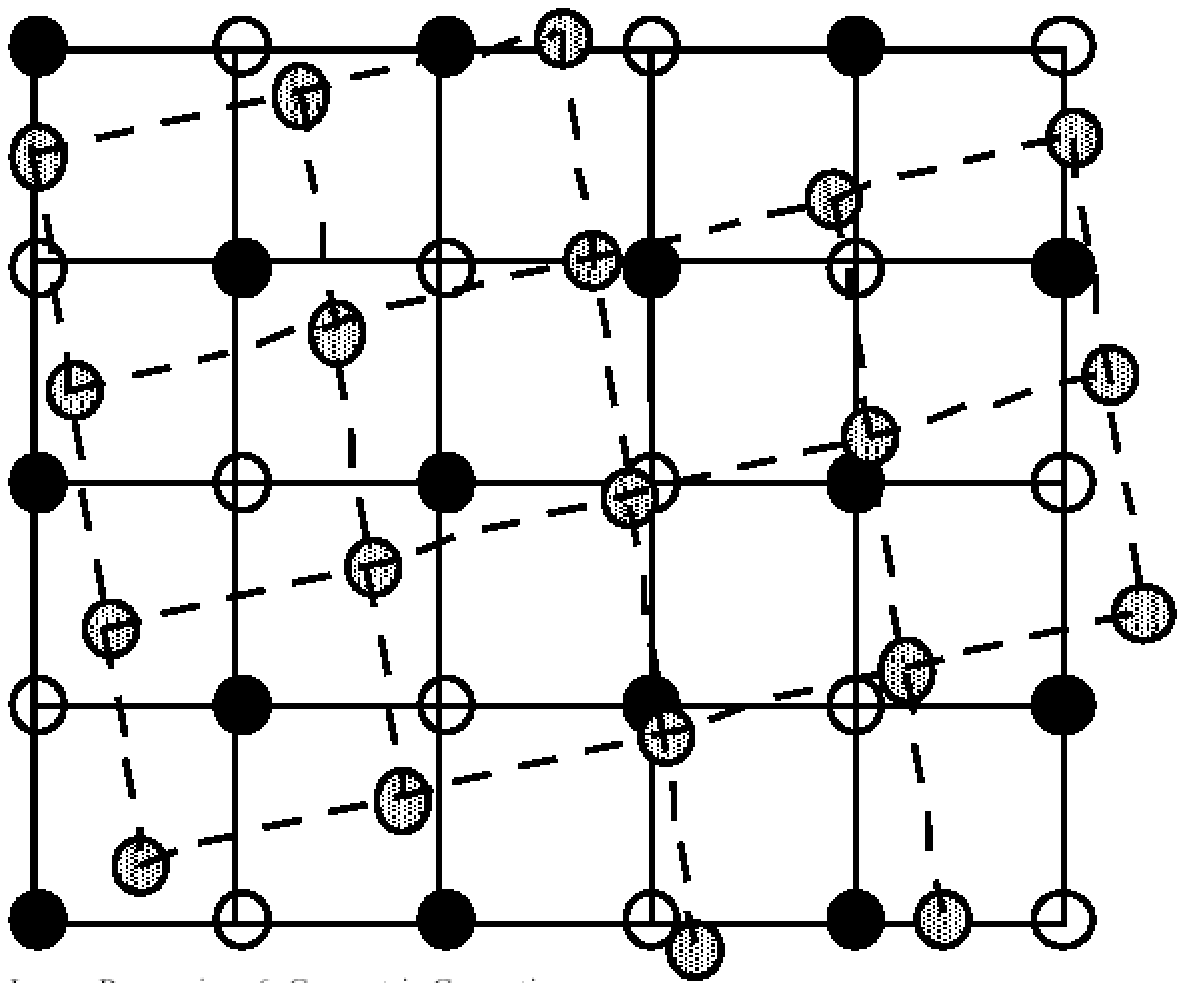
A process in which each data point (pixel) in the base map coordinate system is assigned a value (intensity, gray value, etc.) based on the gray values of local image pixels.

Consider an example in which an image of a checkerboard pattern is geometrically corrected.

Resampling

pixels in the original image data (checkerboard pattern)

desired pixel locations in the base map coordinate system



Nearest-neighbor Resampling

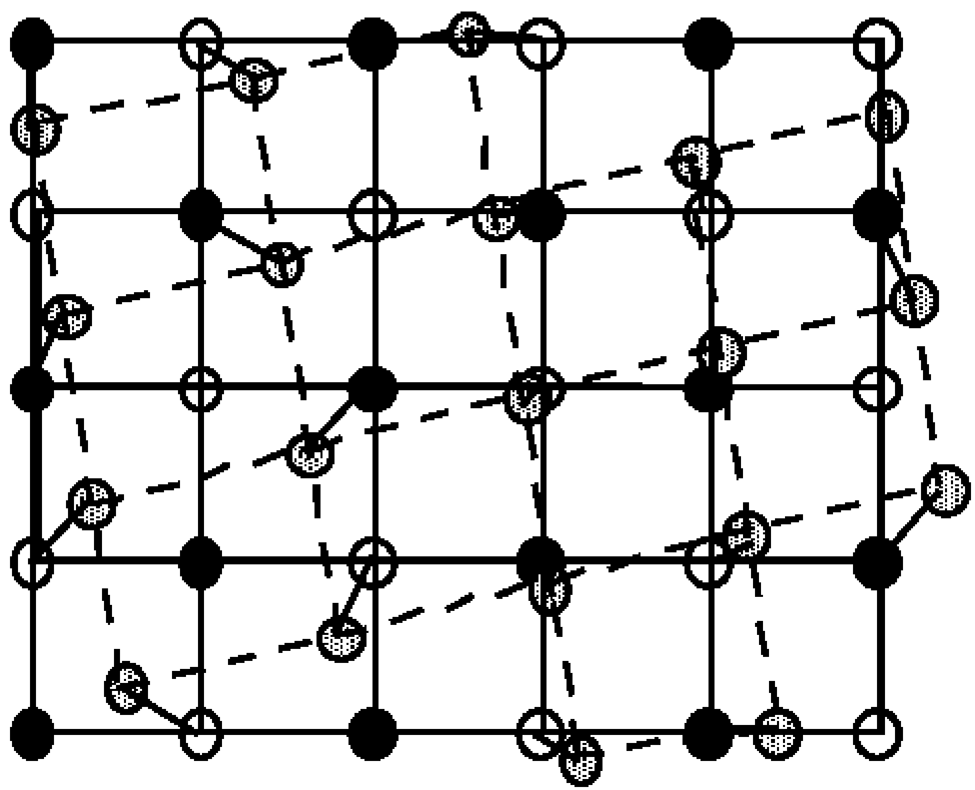
The gray value of the image pixel is assigned to the nearest base map coordinate:

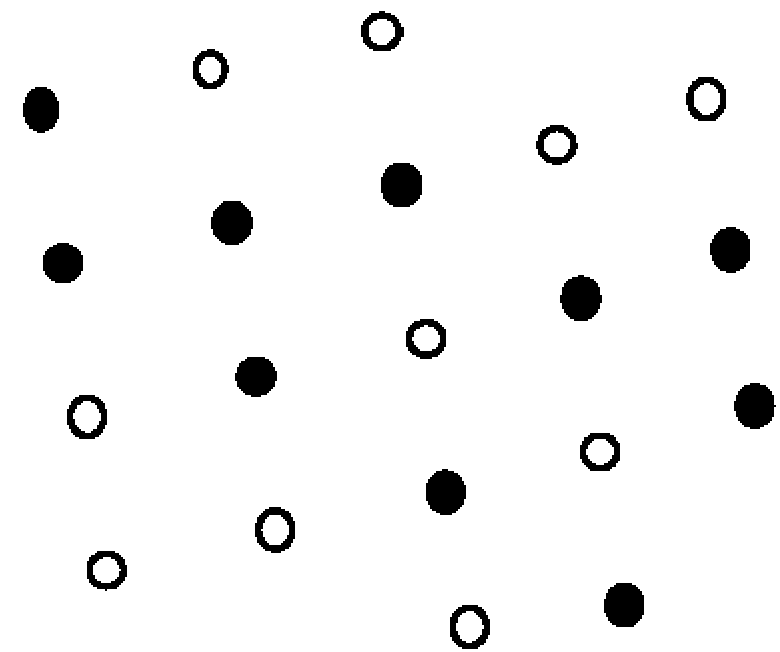
- the value assigned to the grid point will probably not be the same as that which would have been measured at that point.
- tends to result in a blocky appearance at sharp boundaries.
- the base map grid values correspond to actual measured values.
- fastest and cheapest method of resampling.
- does the least radiometric damage to the data.

Nearest-neighbor Resampling

pixels in the original image data (checkerboard pattern)

desired pixel locations in the base map coordinate system



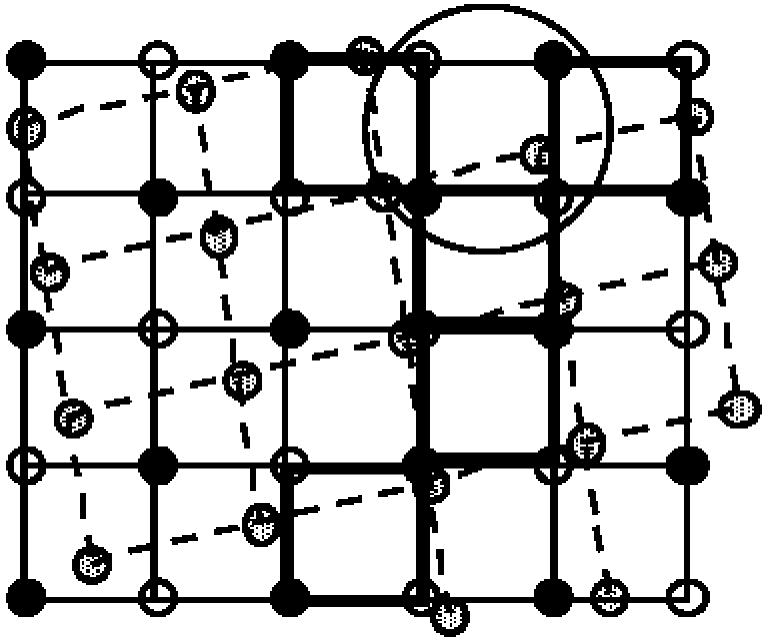


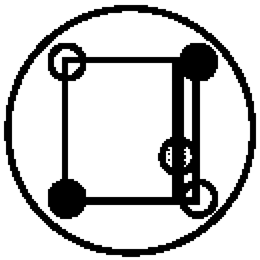
Bilinear Interpolation

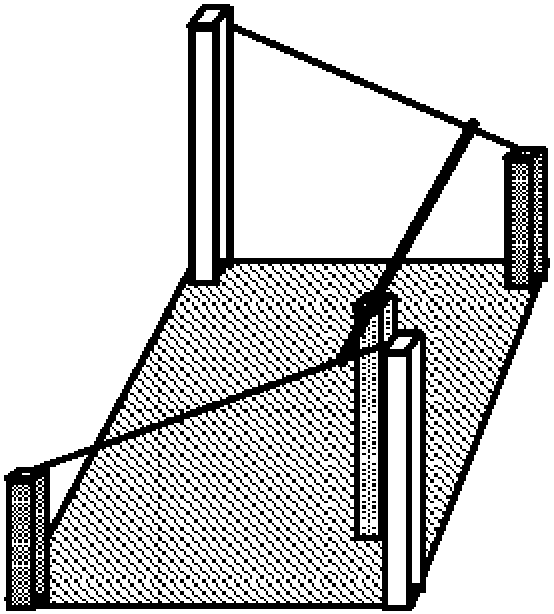
Two-dimensional linear interpolation (3 operations)
Uses the four nearest neighbors

- the base map grid values correspond to a weighted average of the four nearest neighbors
- relatively fast computation.
- smoothes out the blocky appearance apparent with nearest-neighbor resampling (anti-aliasing).
- increases the effective resolution cell size.

Bilinear Interpolation

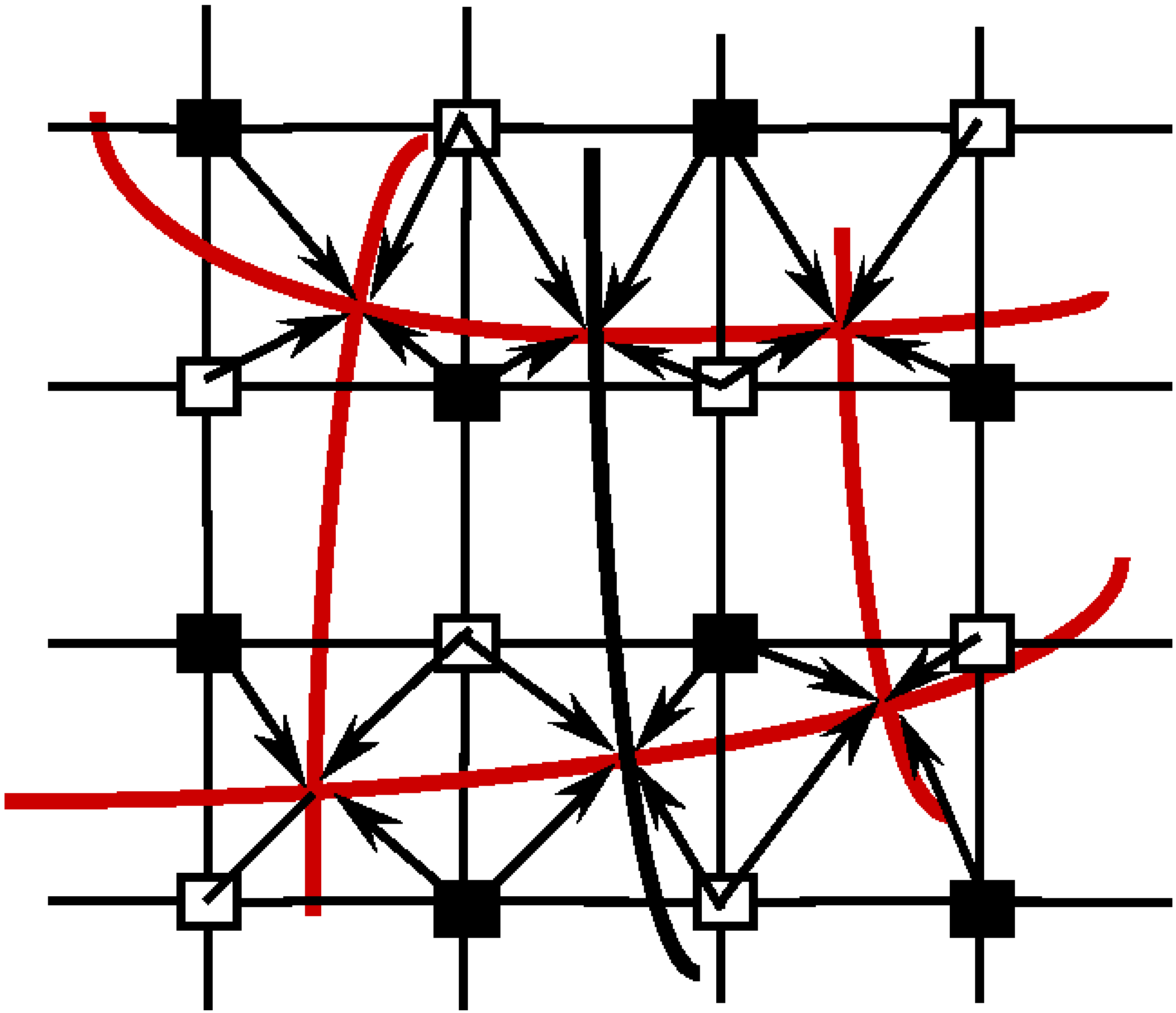






- The contribution of a pixel is inversely proportional to it's distance from the resampled pixel.
- Example **above** illustrates *undersampling*.
- Dark squares are sets of original pixels for which there are no corresponding resampled pixels.

Bilinear Resampling

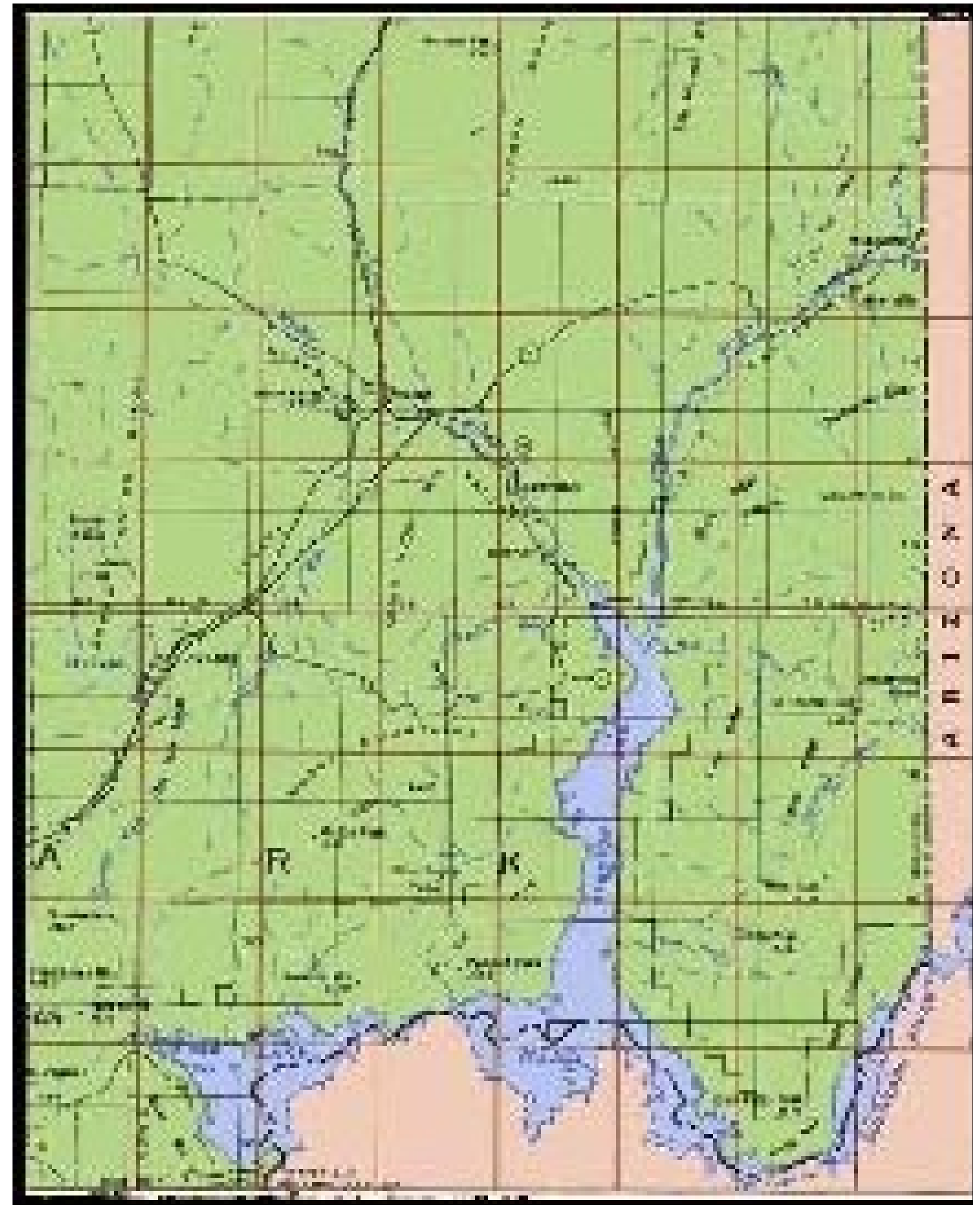


Bicubic Interpolation

Two-dimensional cubic interpolation (3 operations)
Uses the 16 nearest neighbors

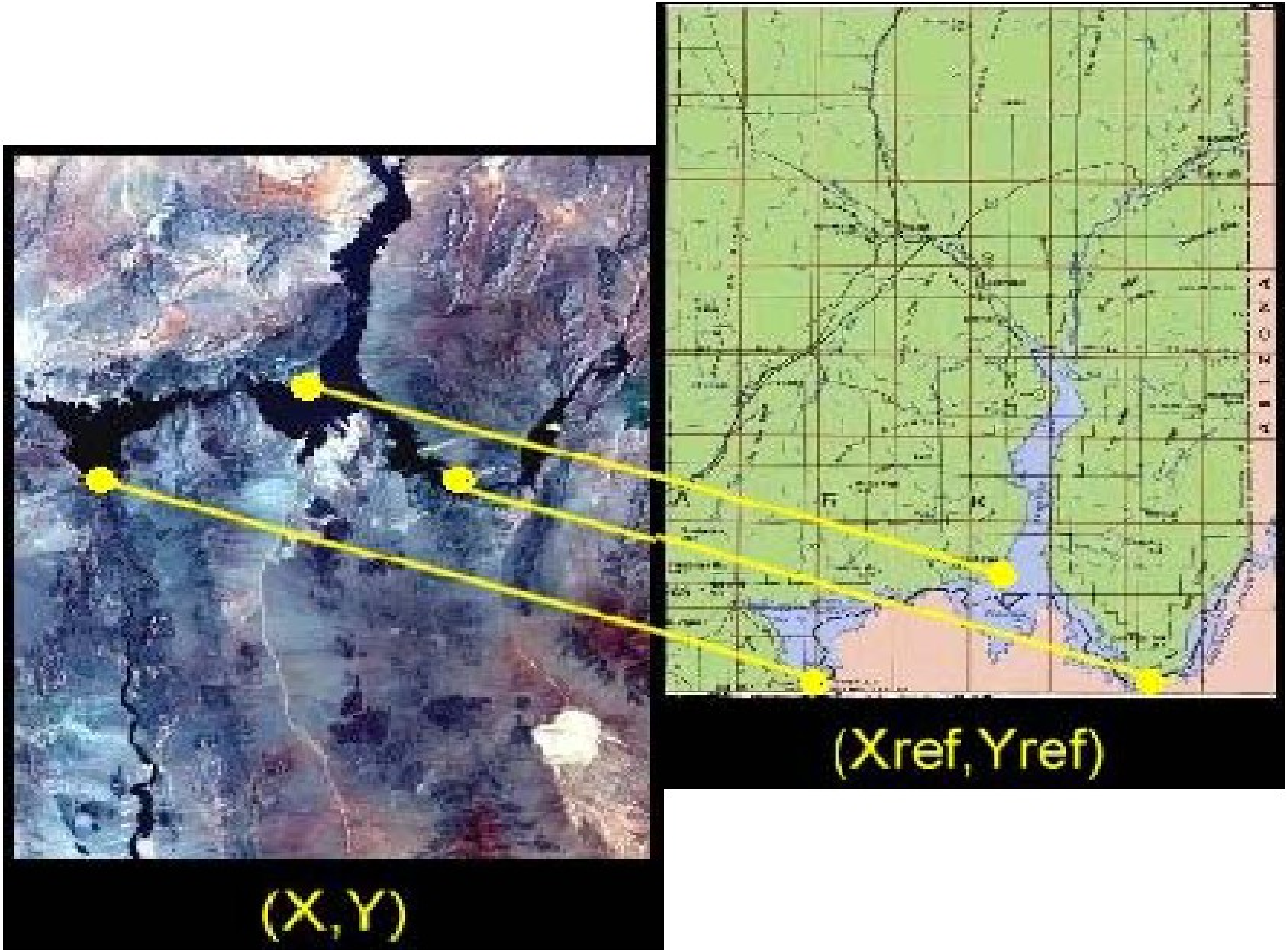
- the base map grid values correspond to a weighted average of the sixteen nearest neighbors
- (a lot) more computation than bilinear interpolation.
- smoothes out the blocky appearance apparent with nearest-neighbor resampling (anti-aliasing).
- further increases the effective resolution cell size.

Geometric Registration: Example, image to map

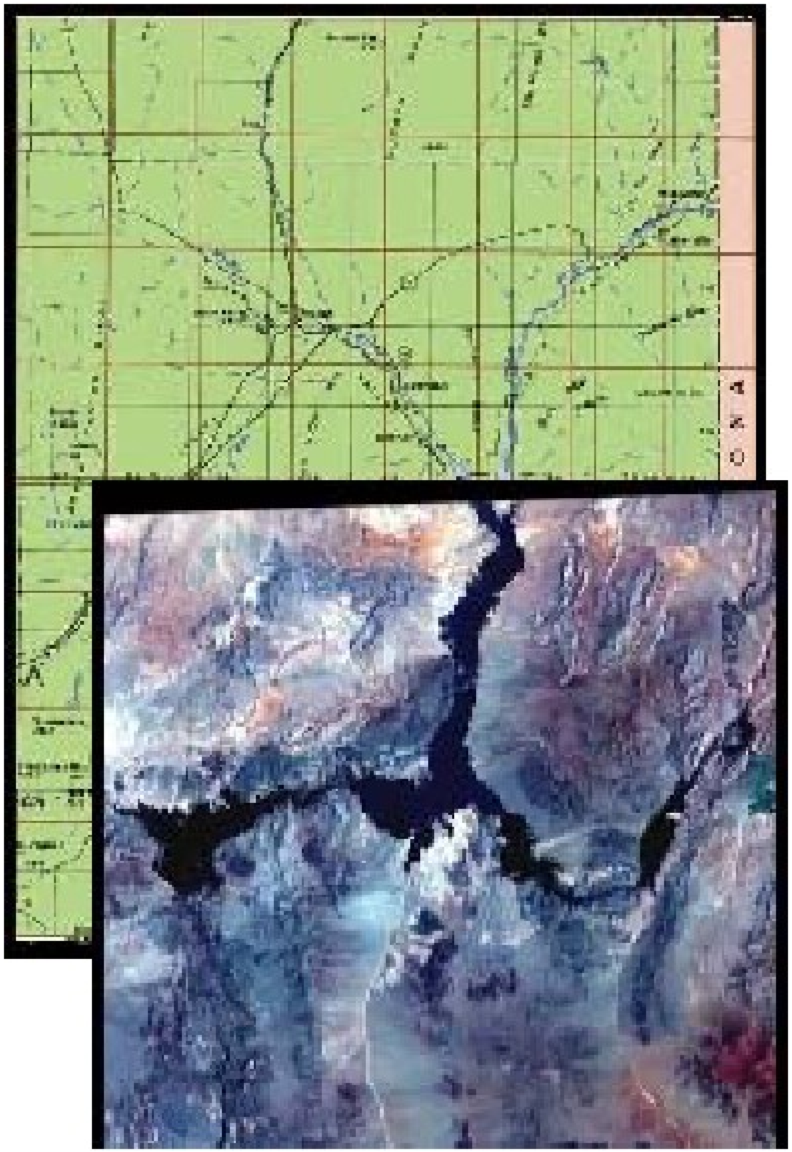


Adapted from: University of Arizona Tutorial on geometric correction

Geometric Registration: selecting GCP's



Geometric Registration: registration



Number of control points: 3

Control points:

Point #	x	y	x'	y'
1	198	157	213	292
2	105	162	120	300
3	260	195	275	325

Linear Transformation:

$$x = -15 + x'$$
$$y = -166 + 0.04\,x' + 1.0783\,y'$$

RMS Error: 1.3610e-11
(at control points)

Is this a reasonable registration?